Variance and Standard Deviation

Variance- Square of the deviation

Standard Deviation- Square root of the variance. It shows how much variation exists from the mean. Symbol = σ

> A low standard deviation indicates that the data points tend to be very close to the mean, whereas high standard deviation indicates that the data points are spread out over a large range of values.

The data from the Ch. 8 Geometry test scores are as follows: 89, 97, 90, 98, 60, 77, 77, 100

Step One: Find the mean

$\mu = $ _____

Step Two: Figure out the deviation. The deviation is _____.

Test Score	Deviation $(x - \mu)$	Square of the Deviation $(x - \mu)^2$
89		
97		
90		
98		
60		
77		
77		
100		
$\mu = $	$\mu = $	$\mu = $

Step Three: Some of these values are negative…How can we make them positive?
Find the _____ OR _____.

Find the square for each deviation for each number.

Step Four: Find the mean of the squares of the deviations or the **VARIANCE**.

Step Five: The square root of the Variance is the standard deviation or σ!

$$\sqrt{\quad\quad} = \sigma = \text{_____}$$

Example: Find the variance and standard deviation for the following data: 12, 15, 17, 19, 21, 11

$\mu =$ _____

Variance = _____

$\sigma =$ _____

Try these on your own:

1. Find the variance and standard deviation for the following data: 3, 4, 5, 6, 7, 8, 9, 10

   $\mu =$ _____

   Variance = _____

   $\sigma =$ _____

2. Find the variance and standard deviation for the following data: 100, 200, 300, 400, 500, 600

   $\mu =$ _____

   Variance = _____

   $\sigma =$ _____

3. Find the variance and standard deviation for the following data: 91, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101

   $\mu =$ _____

   Variance = _____

   $\sigma =$ _____

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The data from the Ch. 8 Geometry test scores are as follows: 89, 97, 90, 98, 60, 77, 77, 100

Step One: Find the mean

$\mu =$ _____ 80 _____

Step Two: Figure out the deviation. The deviation is _____ $X - \mu$ _____ .

Test Score	Deviation $(x - \mu)$	Square of the Deviation $(x - \mu)^2$
89	$89 - 80 = 3$	9
97	$97 - 80 = 11$	121
90	$90 - 80 = 4$	16
98	$98 - 80 = 12$	144
60	$60 - 80 = -26$	676
77	$77 - 80 = -9$	81
77	$77 - 80 = -9$	81
100	$100 - 80 = 14$	196
$\mu = 80$	$\mu = 0$	$\mu = 165.5$ ← variance

Step Three: Some of these values are negative…How can we make them positive?
Find the __absolute value__ OR __square them__ .

> Find the square for each deviation for each number.

Step Four: Find the mean of the squares of the deviations or the **VARIANCE**.

Step Five: The square root of the Variance is the standard deviation or σ!

$$\sqrt{165.5} = \sigma = 12.86$$

Example: Find the variance and standard deviation for the following data: 12, 15, 17, 19, 21, 11

$\mu = $ 15.83

Variance = 12.8

$\sigma = $ 3.58

Deviations: squared

12 - 15.83 = -3.83 = 14.67
15 - 15.83 = -0.83 = 0.69
17 - 15.83 = 1.17 = 1.37
19 - 15.83 = 3.17 = 10.05
21 - 15.83 = 5.17 = 26.73
11 - 15.83 = -4.83 = 23.33

Try these on your own:

1. Find the variance and standard deviation for the following data: 3, 4, 5, 6, 7, 8, 9, 10

$\mu = $ 6.5

Variance = 5.25

$\sigma = $ 2.29

3 - 6.5 = -3.5 = 12.25
4 - 6.5 = -2.5 = 6.25
5 - 6.5 = -1.5 = 2.25
6 - 6.5 = -0.5 = 0.25
7 - 6.5 = 0.5 = 0.25
8 - 6.5 = 1.5 = 2.25

9 - 6.5 = 2.5 = 6.25
10 - 6.5 = 3.5 = 12.25

2. Find the variance and standard deviation for the following data: 100, 200, 300, 400, 500, 600

$\mu = $ 350

Variance = 29166.67

$\sigma = $ 170.78

100 - 350 = -250 = 62500
200 - 350 = -150 = 22500
300 - 350 = -50 = 2500
400 - 350 = 50 = 2500
500 - 350 = 150 = 22500
600 - 350 = 250 = 62500

3. Find the variance and standard deviation for the following data: 91, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101

$\mu = $ 95.58

Variance = 11.08

$\sigma = $ 3.33

91 - 95.58 = -4.58 = 20.98
91 - 95.58 = -4.58 = 20.98
92 - 95.58 = -3.58 = 12.82
93 - 95.58 = -2.58 = 6.66
94 - 95.58 = -1.58 = 2.5
95 - 95.58 = -0.58 = 0.34
96 - 95.58 = 0.42 = 0.18
97 - 95.58 = 1.42 = 2.02
98 - 95.58 = 2.42 = 5.86
99 - 95.58 = 3.42 = 11.7
100 - 95.58 = 4.42 = 19.54
101 - 95.58 = 5.42 = 29.38

Z-scores

Z-score- Indicates how many standard deviations a data value is above or below the mean. The process of finding the z-scores is called standardizing or normalizing.

$$z = \frac{x - \mu}{\sigma}$$

(handwritten annotations: "# data" pointing to x, "mean" pointing to μ, "standard deviation" pointing to σ)

The data from the Ch. 8 Geometry test scores are as follows: 89, 97, 90, 98, 60, 77, 77, 100

What is the z-score for an 89?

Step One: Find the mean.

$\mu = \underline{86}$

Step Two: Find the standard deviation. ✱ Notice this is in the calculator! Labeled σx

$\sigma = \underline{12.86}$

Step Three: Find the z-score

$$z = \frac{x - \mu}{\sigma} \qquad \frac{89 - 86}{12.86}$$

$z = \underline{0.23}$

What is the z-score for a 60?

$$\frac{60 - 86}{12.86}$$

$z = \underline{-2.02}$

What is the z-score for a 100?

$$\frac{100 - 86}{12.86}$$

$z = \underline{1.09}$

Amy took the ACT and got a score of 27 with a mean of 21 and a standard deviation of 5.3. Stephanie took the SAT and got a score of 660 with a mean of 515 and a standard deviation of 116. Which student scored higher?

Amy's z-score
$$\frac{27 - 21}{5.3} = 1.13$$

Stephanie's z-score
$$\frac{660 - 515}{116} = 1.25$$

Stephanie scored higher!